UNIVERSITY OF MYSORE
Department of Studies in Mathematics,
Manasagangotri, Mysore-570 006.

Course: M.Sc., in Mathematics

PAPER TITLES

	TITLE OF THE PAPER
I Semester:	M101 Algebra – I M102 Linear Algebra M103 Real Analysis – I M104 Real Analysis – II M105 Complex Analysis – I
II Semester:	M201 Algebra – II M202 Real Analysis – III M203 Complex Analysis – II M204 Topology – I M205 Combinatorics and Graph Theory
III Semester:	M301 "Choice Based Paper" M302 Commutative Algebra M303 Elements of Functional Analysis M304 Topology – II M305 Special Subject – I
IV Semester:	M401 Differential Geometry M402 Measure and Integration M403 Ordinary and Partial Differential Equations M404 Special Subject – II M405 Special Subject – III

Scheme of Examination under Choice Based Scheme

Semester	Code No.	Title of the Paper	Teaching hours/ Week	Distribution of Marks				Total Marks
				Theory Exam.	Internal Assessment			
					Test	Assignment	Seminar	
I	M 101	Algebra-I	4	80	15	5	-	100
	M 102	Linear Algebra	4	80	15	5	-	100
	M 103	Real Analysis-I	4	80	15	5	-	100
	M 104	Real Analysis-II	4	80	15	5	-	100
	M 105	Complex Analysis-I	4	80	15	5	-	100

II	M 201	Algebra-II	4	80	15	5		100
	M 202	Real Analysis-III	4	80	15	5	-	100
	M 203	Complex Analysis-II	4	80	15	5	-	100
	M 204	Topology-I	4	80	15	5	-	100
	M 205	Combinatorics and Graph Theory	4	80	15	5	-	100
III	M 301	**Choice Based Paper**	4	80	15	5	-	100
	M 302	Commutative Algebra	4	80	15	5	-	100
	M 303	Elements of Functional Analysis	4	80	15	5	-	100
	M 304	Topology-II	4	80	15	5	-	100
	M 305	Special Subject-I	4	80	15	5	-	100
IV	M 401	Differential Geometry	4	80	10	5	5	100
	M 402	Measure and Integration	4	80	10	5	5	100
	M 403	Ordinary and Partial Differential Equations	4	80	10	5	5	100
	M 404	Special Subject-II	4	80	10	5	5	100
	M 405	Special Subject-III	4	80	10	5	5	100

Total Marks: 2,000

Syllabus

Annexure I

“Choice Based Paper” offered at the Department of Studies in Mathemeatics:

Title: DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS:

Recap of Elementary Functions of Calculus - Properties of limits, derivatives and integrals of elementary functions of Calculus, Polynomials, Rational functions, exponential and logarithmic functions, trigonometric and inverse trigonometric functions, hyperbolic and inverse hyperbolic functions.

Special Functions of Mathematical Physics - Bessel functions, Legendre polynomials, Tchebyshev polynomials, Hermite polynomials and Laguerre polynomials. Power series solutions of Second Order Linear Differential Equations. Their Mathematical properties.

Applications of First Order Ordinary Differential Equations - Simple problems of dynamics – falling bodies and other motion problems, Simple problems of Chemical reactions and mixing, Simple problems of growth and decay.

Applications of Second Order Ordinary Differential Equations - Undamped simple harmonic motion, damped vibrations, Forced vibrations, Problems on simple electric circuits – Laplace transforms.

Partial Differential Equations and Boundary Value Problems - Fourier series solutions of Heat equation, Wave equation and Laplace equation. Dirichlet, Neumann and mixed boundary value problems.

References:

1. G. F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw-Hill, New Delhi, 1991.
2. E. D. Rainville and P. Bedient – Elementary course on Ordinary Differential Equations, Macmillan, New York, 1972.
3. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol. I, Tata McGraw-Hill, New Delhi, 1975.

Annexure II

FIRST SEMESTER

M 101 Algebra I

Number theory - Congruences, residue classes, theorems of Fermat, Euler and Wilson, linear congruences, elementary arithmetical functions, primitive roots, quadratic residues and the law of quadratic reciprocity.
Groups - Lagrange's Theorem, homomorphism and isomorphism, normal subgroups and factor groups, the fundamental theorem of homomorphism, two laws of isomorphism, permutation groups and Cayley's theorem, Sylow's theorems.

References:

1. D. M. Burton – Elementary Number Theory, Tata McGraw-Hill, New Delhi, 6th Ed.,
2. I. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theory of Numbers, New York, John Wiley and Sons, Inc., 2004, 5th Ed.,
3. G. A. Jones and J. M. Jones – Elementary Number Theory, Springer, 1998.
4. Thomas W. Hungerford – Algebra, Springer International Edition, New York.
5. Michael Artin – Algebra, Prentice-Hall of India, New Delhi.
6. J. A. Gallian – Contemporary Abstract Algebra, Narosa Publishing House, 4th Ed.,
7. D. S. Dummit and R. M. Foote – Abstract Algebra, John Wiley and Sons, 1999.
8. I. N. Herstein – Topics in Algebra, Vikas Publishing House, New Delhi.
9. J. B. Fraleigh – A First course in Abstract Algebra, Addison-Wesley,
10. N. S. Gopalakrishnan – University Algebra, New Age International, 2nd Ed.

M 102 Linear Algebra

Vector spaces, subspaces, quotient spaces, linear independence, bases, dimension, the algebra of linear transformations, kernel, range, isomorphism, matrix representation of a linear transformation, change of bases.
Linear functionals, dual space, projection. Determinant function, eigen values and eigen vectors, Cayley-Hamilton theorem.
Invariant sub-spaces and canonical forms - diagonal form, triangular form, Jordon form, Inner product spaces - Real quadratic forms, Sylvester's law of inertia.

References:

1. Jimmie Gilbert and Linda Gilbert – Linear Algebra and Matrix Theory, Academic Press, An imprint of Elsevier.
2. I. N. Herstein – Topics in Algebra, Vikas Publishing House, New Delhi.
3. Hoffman and Kunze – Linear Algebra, Prentice-Hall of India, 1978, 2nd Ed.,
4. P. R. Halmos – Finite Dimensional Vector Space, D. Van Nostrand, 1958.
5. S. Kumeresan – Linear Algebra, A Geometric approach, Prentice Hall India, 2000.

M 103 Real Analysis I

The extended real number system, the n-dimensional Euclidean space, the binomial inequality, the inequality of the arithmetic and geometric means, the inequality of the power means, Cauchy's, Holder's inequality and Minkowski's inequality.
Numerical sequences and series of real numbers, convergent sequences, Cauchy sequences, upper and lower limits, series of non-negative terms, the number `e', tests of convergence, multiplications of series, re-arrangements. Double series, infinite products.

References:

1. W. Rudin – Principles of Mathematical Analysis, International Student edition, McGraw Hill, 3rd Ed.
2. T. M. Apostal – Mathematical Analysis, Addison Wesley, Narosa, New Delhi, 2nd Ed.
3. R. R. Goldberg – Methods of real Analysis, Oxford and IBH, New Delhi.
4. Torence Tao – Analysis I, Hindustan Book Agency, India, 2006.
5. Torence Tao – Analysis II, Hindustan Book Agency, India, 2006.
6. Kenneth A. Ross – Elementary Analysis: The Theory of Calculus, Springer International Edition, 2004.

M 104 Real Analysis II

Finite, countable and uncountable sets, the topology of the real line.
Continuity, uniform continuity, properties of continuous functions, discontinuities, monotonic functions.
Differentiability, mean value theorems, L' Hospital rule, Taylor's theorem, maxima and minima.
The Riemann-Stieltje's integral, criterion for integrability. Properties of the integral, classes of integrable functions. The integral as the limit of a sum. First and second mean value theorems. Integration and differentiation. Functions of bounded variation.

References:

1. W. Rudin – Principles of Mathematical Analysis, International Student edition, McGraw-Hill, 3rd Ed..
2. Torence Tao – Analysis I, Hindustan Book Agency, India, 2006.
3. Torence Tao – Analysis II, Hindustan Book Agency, India, 2006.
4. T. M. Apostal – Mathematical Analysis, Addison Wesley, Narosa, 2nd Ed.,
5. R. R. Goldberg – Methods of real Analysis, Oxford and IBH Publishing Company, New Delhi.
6. Kenneth A. Ross – Elementary Analysis: The Theory of Calculus, Springer International Edition, 2004.

M 105 Complex Analysis I

Algebra of complex numbers, geometric representation of complex numbers. Riemann sphere and Stereographic projection, Lines, Circles.
Limits and Continuity, Analytic functions, Cauchy-Riemann equations, Harmonic functions, Polynomials and Rational functions.
Elementary theory of power series - sequences, series, uniform convergence of power series, Abel's limit theorem, The elementary functions.
Topology of the complex plane. Linear fractional transformations, Cross-ratio, Symmetry, Elementary conformal mappings.
Complex integration – Line integrals, Rectifiable arcs, Cauchy's theorem for a rectangle. Cauchy's theorem in a Circular disk, Cauchy's integral formula. Local properties of analytic functions.

References:

1. L. V. Ahlfors – Complex Analysis, McGraw-Hill, Kogakusha, 1979.
2. J. B. Conway – Functions of one complex variable, Narosa, New Delhi.
3. R. P. Boas – Invitation to Complex Analysis, The Random House, 1987
4. B. C. Palka – An Introduction to Complex Function Theory, Springer, 1991.
5. S. Ponnusamy – Foundations of Complex Analysis, Narosa, 1995.

SECOND SEMESTER

M 201 Algebra II

Rings, Integral domains and Fields, Homomorphisms, Ideals and Quotient Rings, Prime and Maximal ideals, Euclidean and principal ideal rings, Polynomials, Zeros of a polynomial, Factorization, Irreducibility criterion.
Adjunction of roots, algebraic and transcendental extensions, Finite fields, Separable and inseparable extensions, Perfect and imperfect fields. Theorem on the primitive element.

References:

1. Thomas W. Hungerford – Algebra, Springer International Edition, New York.
2. Michael Artin – Algebra, Prentice-Hall of India, New Delhi.
3. Joseph A. Gallian – Contemporary Abstract Algebra, Narosa, 4^{th} Ed.,
4. D. S. Dummit and R. M. Foote – Abstract Algebra, John Wiley and Sons,1999, 2^{nd} Ed.,
5. I. N. Herstein – Topics in Algebra, Vikas Publishing House, New Delhi.
6. J. B. Fraleigh – A First course in Abstract Algebra, Addison-Wesley,
7. N. S. Gopalakrishnan – University Algebra, New Age International, 2^{nd} ed.,

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M 202 Real Analysis III

Sequences and series of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Power series, The exponential and logarithmic functions, The trigonometric functions.
Improper integrals and their convergence.
Functions of several variables, partial derivatives, continuity and differentiability, the chain rule, Jacobians, the Implicit function theorem, Taylor's theorem, the Maxima and Minima, Lagrange's multipliers.

References:

1. W. Rudin – Principles of Mathematical Analysis, International Student edition, McGraw-Hill, 3^{rd} Ed.,
2. T.M. Apostal – Mathematical Analysis, Addison Wesley, Narosa, 2^{nd} Ed.,
3. R.R. Goldberg – Methods of Real Analysis, Oxford and IBH, New Delhi.
4. D.V. Widder – Advanced Calculus, Prentice Hall of India, New Delhi, 2^{nd} Ed.,
5. Torence Tao – Analysis I, Hindustan Book Agency, India, 2006.
6. Torence Tao – Analysis II, Hindustan Book Agency, India, 2006.
7. Kenneth A. Ross – Elementary Analysis: The Theory of Calculus, Springer International Edition, 2004.

M 203 Complex Analysis II

The Calculus of Residues – The residue theorem, argument principle, Evaluation of definite integrals.
Harmonic functions – Definition and basic properties, mean value property, Poisson's formula, Schwarz's theorem, reflection principle.
Power series expansions – The Weierstrass theorem, The Taylor series, The Laurent series.
Partial fractions and factorization – Partial fractions, Mittag - Leffer's theorem, Infinite products, Canonical products, The Gamma and Beta functions, Sterling's formula.
Entire functions – Jensen's formula, Hadamard's theorem.

References:

1. L. V. Ahlfors – Complex Analysis, McGraw-Hill, Kogakusha, 1979.
2. J. B. Conway – Functions of one complex variable, Narosa, New Delhi.
3. R. P. Boas – Invitation to Complex Analysis, The Random House, 1987.
4. B. C. Palka – An Introduction to the Complex Function Theory, Springer, 1991.
5. S. Ponnusamy – Foundations of Complex Analysis, Narosa, 1995.

M 204 TOPOLOGY I

Set theoretic preliminaries.
Topological spaces and continuous maps - topological spaces, basis for a topology, the order topology, the product topology on X x X, the subspace topology, closed sets and limit points, continuous functions, the product topology, the metric topology, the quotient topology,.
Connectedness and compactness - connected spaces, connected sets on the real line, path connectedness, compact spaces, compact sets on the line, limit point compactness, local compactness.

References:

1. J. R. Munkres – A First Course in Topology, Prentice Hall India, 2000, 2nd Ed.,
2. G. F. Simmons – Introduction to Topology and Modern Analysis, McGraw-Hill, Kogakusha, 1968.
3. S. Willard – General Topology, Addison Wesley, New York, 1968.
4. J. Dugundji – Topology, Allyn and Bacon, Boston, 1966.
5. J. L. Kelley – General Topology, Van Nostrand and Reinhold Co., New York, 1955.

M 205 COMBINATORICS AND GRAPH THEORY

Partially ordered sets, Lattices, Complete lattices, Distributive lattices, Complements, Boolean Algebra, Boolean expressions, Application to switching circuits, Permutations and Combinations, Pigeon-hole principle, Principle of inclusion and exclusion.

Graphs, Vertices of graphs, Walks and connectedness, Degrees, Operations on graphs, Blocks, Cutpoints, bridges and blocks, Block graphs and Cutpoint graphs, Trees, Elementary properties of trees, Center, Connectivity, Connectivity and line connectivity, Menger's theorem, Partitions, Coverings, Coverings and independence number.

References:

1. C. L. Liu – Elements of Discrete Mathematics, McGraw-Hill, 1986.
2. Kenneth H. Rosen – Discrete Mathematics and its Applications, McGraw-Hill, 2002.
3. F. Harary – Graph Theory, Addition Wesley Reading Mass, 1969.
4. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India, 1987.
5. K. R. Parthasarathy – Basic Graph Theory, Tata McGraw-Hill, New Delhi, 1994.
6. G. Chartand and L. Lesniak – Graphs and Diagraphs, wadsworth and Brooks, 2nd Ed.,
7. Clark and D. A. Holton – A First Look at Graph Theory, Allied publishers.
8. D. B. West – Introduction to Graph Theory, Pearson Education Inc.,2001, 2nd Ed.,
9. J. A. Bondy and U. S. R. Murthy – Graph Theory with applications, Elsevier, 1976.

THIRD SEMESTER

M 301 <u>Choice Based Paper: DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS:</u>

(Offered to the students of other Departments)

Recap of Elementary Functions of Calculus - Properties of limits, derivatives and integrals of elementary functions of Calculus, Polynomials, Rational functions, exponential and logarithmic functions, trigonometric and inverse trigonometric functions, hyperbolic and inverse hyperbolic functions.

Special Functions of Mathematical Physics - Bessel functions, Legendre polynomials, Tchebyshev polynomials, Hermite polynomials and Laguerre polynomials. Power series solutions of Second Order Linear Differential Equations. Their Mathematical properties.

Applications of First Order Ordinary Differential Equations - Simple problems of dynamics – falling bodies and other motion problems, Simple problems of Chemical reactions and mixing, Simple problems of growth and decay.

Applications of Second Order Ordinary Differential Equations - Undamped simple harmonic motion, damped vibrations, Forced vibrations, Problems on simple electric circuits – Laplace transforms.

Partial Differential Equations and Boundary Value Problems - Fourier series solutions of Heat equation, Wave equation and Laplace equation. Dirichlet, Neumann and mixed boundary value problems.

References:

1. G. F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw-Hill, New Delhi, 1991.
2. E. D. Rainville and P. Bedient – Elementary course on Ordinary Differential Equations, Macmillan, New York, 1972.
3. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol. I, Tata McGraw-Hill, New Delhi, 1975.

M 302 COMMUTATIVE ALGEBRA

Rings and ideals - Rings and ring homomorphisms, Ideals, Quotient rings, zero-divisors, nilpotent elements, units, prime ideals and maximal ideals, the prime spectrum of a ring, the nil radical and Jacobson radical, operation on ideals, extension and contraction.
Modules - Modules and modules homomorphisms, submodules and quotient modules, Direct sums, Free modules Finitely generated modules, Nakayama Lemma, Simple modules, Exact sequences of modules.
Modules with chain conditions - Artinian and Noetherian modules, modules of finite length, Artnian rings, Noetherian rings, Hilbert basis theorem.

References:

1. M. F. Atiyah and I. G. Macdonald – Introduction to Commutative Algebra, Addison-Wesley.
2. C. Musili – Introduction to Rings and Modules, Narosa Publishing House.
3. Miles Reid – Under-graduate Commutative Algebra, Cambridge University Press.
4. N. S. Gopalakrishnan, Commutative Algebra, Oxonian Press.

M 303 ELEMENTS OF FUNCTIONAL ANALYSIS

Metric completion. Banach's contraction mapping theorem and applications, Baire' category theorem, Ascoli - Arzela theorem.
Linear spaces and linear operators, Norm of a bounded operator, The Hahn - Banach extension theorem, Stone - Weirstrass theorem.
Open mapping and Closed Graph theorems. The Banach - Steinhaus Principle of Uniform Boundedness.
Hilbert spaces- The orthogonal projection, Nearly orthogonal elements, Riesz's lemma, Riesz's representation theorem.

References:

1. G. F. Simmons – Introduction to Topology and Modern Analysis, Tata McGraw-Hill, New Delhi.
2. A. E. Taylor – Introduction to Functional Analysis, Wiley, New York, 1958.
3. A. Page and A. L. Brown – Elements of Functional Analysis.
4. George Bachman and Lawrence Narici – Functional Analysis, Dover Publications, Inc., Mineola, New York.
5. J. B. Conway – A Course in Functional Analysis, GTM, Vol. 96., Springer, 1985.

M 304 TOPOLOGY II

Countability and Separation axioms - the countability axioms, the separation axioms, normality of a compact Hausdorff space. Urysohn's lemma, Tietze's extension theorem, Urysohn's metrization theorem, Partitions of unity, Tychonoff's theorem on the product of compact spaces.
Local finiteness, Paracompactness, Normality of a paracompact space.
The Fundamental group and the Fundamental group of a circle, The Fundamental group of the punctured plane, Essential and Inessential Maps, The Fundamental Theorem of Algebra.

References:

1. James R. Munkres - A First Course in Topology , Prentice Hall India, 2000, 2nd Ed.,
2. G. F. Simmons – Introduction to Topology and Modern Analysis, McGraw-Hill, Kogakusha, 1968.
3. S. Willard – General Topology, Addison Wesley, New York, 1968.
4. J. Dugundji – Topology, Allyn and Bacon, Boston, 1966.
5. J. L. Kelley – General Topology, Van Nostrand and Reinhold Co., New York, 1955.

SPECIAL PAPER
M 305 – Special Subject - I
(Students can opt any ONE of the following papers)

(1) GALOIS THEORY

Algebraically closed fields and algebraic closures, The existence of an algebraic closure, The basic isomorphisms of algebraic field theory, Automorphisms and fixed fields, The Frobenius automorphism, The isomorphism extension theorem, The index of a field extension, Splitting fields, Separable extensions, Perfect fields, Normal extensions. Galois theory - the main theorem of Galois theory, Galois groups over finite fields, Symmetric functions, Cyclotomic extensions, Constructible numbers, the impossibility of certain geometrical constructions, constructible polygons, Subnormal and normal series, the Jordan - Holder theorem, Radical extensions and solution of equation by radicals, The insolvability of the quintic.

References:

1. J. B. Fraleigh – A First Course in Abstract Algebra, Narosa Publishing House.
2. Ian Steward – Galois Theory, Chapman and Hall.
3. Joseph Rotman – Galois Theory, Universitext Springer, 1998.
4. Michael Artin – Algebra, Prentice-Hall of India, New Delhi.
5. Joseph A. Gallian – Contemporary Abstract Algebra, Narosa Publishing House, 4th Ed.,
6. D. S. Dummit and R. M. Foote – Abstract Algebra, John Wiley and Sons, 1999.
7. I. N. Herstein – Topics in Algebra, Vikas Publishing House, New Delhi.
8. N. S. Gopalakrishnan – University Algebra, New Age International, 2nd Ed.,

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(2) THEORY OF NUMBERS

Prime numbers, The Fundamental theorem of Arithmetic, The series of Reciprocals of primes, The Euclidean Algorithm.
Arithmetical Functions – The Mobius function, The Euler' function and Sigma function, The Dirichlet product of Arithmetical functions, Multiplicative functions.
Averages of Arithmetical functions – Euler summation formula, Some elementary asymptotic formulas, The average orders of $d(n)$, $\sigma(n)$, $\phi(n)$, $\mu(n)$, An application to the distribution of lattice points visible from the origin.
Fermat and Mersenne numbers.
Farey series, Farey dissection of the continuum,
Irrational numbers-Irrationality of m^{th} root of N, e and π
Approximation Irrational numbers, Hurwitz's Theorem, Representation of a number by two or four squares, Definition g(k) and G(k), Proof of g(4)<50, Perfect numbers.

Continued fractions - Finite continued fractions, Convergent of a continued fraction, Continued fractions with positive quotients. Simple continued fractions, The representation of an irreducible rational fraction by a simple continued fraction. The continued fraction algorithm and Euclid's algorithm. The difference between the fraction and its convergents, Infinite simple continued fractions, the representation of an irrational number by an infinite continued fraction, Equivalent numbers and periodic continued fractions, some special quadratic surds, and the series of Fibonacci and Lucas.

References:

1. G. H. Hardy and E. M. Wright – An Introduction to Theory of Numbers, Oxford University Press, 1979, 5th Ed.,
2. I. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theory of Numbers, New York, John Wiley and Sons, Inc., 2004, 5th Ed.,
3. Bruce C. Berndt – Ramanujan's Note Books Volume-1 to 5, Springer.
4. G. E. Andrews – Number Theory, Dover Books, 1995.
5. T. M. Apostol – Introduction to Analytic Number Theory, Narosa Publishing House, New Delhi.

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(3) ALGORITHMS AND COMPUTATIONS

Introduction to Computers, Flowcharts, Algorithms and their features, Languages, Types of language and translators.
Numerical Algorithms - Solving a simultaneous system of linear equations using interactive and direct methods.
Interpolation algorithms - equal, unequal intervals, central difference and inverse interpolation.
Numerical differential and integration and their errors calculations.
Graph theoretical algorithms - Connectivity, finding shortest path between two vertices, enumeration of all paths, construction of minimum spanning tree, cutest, cut vertex, coding and decoding.
Computation - Algorithms complexities, strategies, Divide and conquer, greedy technique, Introduction to NP hard problems.

References:

1. Conte and D’bear – Numerical Algorithms, McGraw-Hill, 1985.
2. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India, 1987.
3. E. V. Krishnamurthy – Introductory Theory of Computer Science, Prentice Hall of India, 1980.
4. Horowitz and Sahni – Fundamentals of Computer Algorithms, Addison Wesley, 1987.
5. V. Rajaraman – Computer Oriented Numerical Methods, Prentice Hall of India, 1980.
6. G. Shankar Rao – Numerical Analysis, Prentice Hall of India, 1985.

FOURTH SEMESTER

M 401 DIFFERENTIAL GEOMETRY

Plane curves and Space curves – Frenet-Serret Formulae.
Global properties of curves – Simple closed curves, The isoperimetric inequality, The Four Vertex theorem.
Surfaces in three dimensions – Smooth surfaces, Tangents, Normals and Orientability, Quadric surfaces.
The First Fundamental form – The lengths of curves on surfaces, Isometries of surfaces, Conformal mappings of surfaces, Surface area, Equiareal Maps and a theorem of Archimedes.
Curvature of surfaces – The Second Fundamental form, The Curvature of curves on a surface, Normal and Principal Curvatures.
Gaussian Curvature and The Gauss' Map – The Gaussian and The mean Curvatures, The Pseudo sphere, Flat surfaces, Surfaces of Constant Mean Curvature, Gaussian Curvature of Compact surfaces, The Gauss' Map.
Geodesics – Definition and Basic properties, Geodesic equations, Geodesics on surfaces of revolution, Geodesics as shortest paths, Geodesic co-ordinates.

References:

1. A. Pressley – Elementary Differential Geometry, Under-graduate Mathematics Series, Springer.
2. T. J. Willmore – An Introduction to Differential Geometry, Oxford University Press.
3. D. Somasundaram – Differential Geometry: A First Course, Narosa, 2005.

M 402 MEASURE AND INTEGRATION

Lebesgue measure - outer measure, measurable sets and Lebesgue measure, a non-measurable set, measurable functions.
The Lebesgue integral – the Lebesgue Integral of a bounded function over as set of finite measure, the integral of a non-negative function, the general Lebesgue integral.
Differentiation and integration - Differentiation of monotonic functions, functions of bounded variation, differentiation of an integral, absolute continuity.
Measure and integration - Measure spaces, Measurable functions, integration, Signed measures, the Radon - Nikodym theorem, Measure and outer measure, outer measure and measurability, the extension theorem, product measures.

References:

1. H. L. Royden – Real Analysis, Prentice Hall, 3rd Ed.,
2. G. de Barra – Measure Theory and Integration, Wiley Eastern Limited.
3. Inder K. Rana – An Introduction to Measure and Integration, Narosa, 1997.

M 403 ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Linear Second Order Equations - Initial value problem, Existence and Uniqueness by Picard's Theorem, Wronskian, separation and comparison theorems, Poincare phase plane, variation of parameters.
Boundary value problems – Sturm - Liouvile system, eigen values and eigen functions, simple properties, expansion in eigen functions, Parseval's identity, Green's function method.
Power series solutions - Solution near ordinary and regular singular point. Convergence of the formal power series, applications to Legendre, Bessel, Hermite, Laguerre and hypergeometric differential equations with their properties.
Partial differential equations - Cauchy problems and characteristics, Classification of Second order PDE's, reduction to canonical forms, derivation of the equations of mathematical physics and their solutions by separation of variables.

References:

1. E. A. Coddington and N. Levinson – Theory of Ordinary Differential equations, Tata McGraw-Hill, New Delhi.
2. R. Courant and D. Hilbert – Methods of Mathermatical Physics, Vol. I. & II, Tata McGraw-Hill, New Delhi, 1975.
3. G. F. Simmons – Differential Equations with applications and Historical Notes, Tata McGraw-Hill, New Delhi, 1991.
4. I. N. Sneddon – Theory of Partial differential equations, McGraw-Hill, International Student Edition.
5. S. G. Deo and V. Raghavendra – Ordinary Differential Equations and Stability Theory, Tata McGraw-Hill, New Delhi.

SPECIAL PAPERS: <u>M 404 - II</u> AND <u>M 405 - III</u>
(Students can opt any TWO of the following papers)

(1) ALGEBRAIC NUMBER THEORY

Number theoretical applications of unique factorization - Algebraic integers, Quadratic fields, Certain Euclidean rings of algebraic integers, Some Diophantine equations, Ramanujan - Nagell theorem.
Factorization of Ideals - Dedekind domains, Fractional ideals, Invertible ideals, Prime factorization of ideals, Class group and Class number, Finiteness of the Class group, Class number computations.

References:

1. Karlheinz Spindler – Abstract Algebra with Applications, Vol. II, Rings and Fields, Marcel Dekker, Inc.
2. I. N. Stewart and David Tall – Algebraic Number Theory, Chapman and Hall.
3. Jody Esmonde and M. Ram Murthy – Problems in Algebraic Number Theory, Springer Verlag.
4. I. S. Luthar and I. B. S. Passi – Algebra Vol. II: Rings, Narosa Publishing House.

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(2) THEORY OF PARTITIONS

Partitions - partitions of numbers, the generating function of p(n), other generating functions, two theorems of Euler, Jacobi's triple product identity and its applications, ${}_1\Psi_1$ - summation formula and its applications, combinatorial proofs of Euler's identity, Euler's pentagonal number theorem, Franklin's combinatorial proof, congruence properties of partition function, the Rogers - Ramanujan Identities.
Elementary series - product identities, Euler's, Gauss', Heine's, Jacobi's identities.
Restricted Partitions – Gaussian, Frobinius partitions.

References:

1. G. H. Hardy and E. M. Wright – An Introduction to Theory of Numbers, Oxford University Press, 1979, 5^{th} Ed.,
2. I. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theory of Numbers, New York, John Wiley and Sons, Inc., 2004, 5^{th} Ed.,
5. Bruce C. Berndt – Ramanujan's Note Books Volumes-1 to 5.
6. G. E. Andrews – The Theory of Partitions, Addison Wesley, 1976.
7. A. K. Agarwal, Padmavathamma, M. V. Subbarao – Partition Theory, Atma Ram & Sons, Chandigarh, 2005.

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(3) LIE ALGEBRA

Definition and example, construction of Lie and associative algebras, Linear Lie algebras. Derivations, inner derivations of Lie algebras. Determination of the Lie algebras of low dimensionalities. Representations and modules, some basic module operations, ideals and homomorphisms, Fundamental thorem of homomorphisms, Solvable Lie algebras, properties of solvable Lie algebras, Nilpotent Lie algebras, properties of nilpotent Lie algebras, Engle's theorem, Lie's theorem, Cartan's criterion for solvability.

References:

1. James E. Humphreys – Introduction to Lie algebras and representation theory, Springer Verlag, New Delhi.
2. N. Jacobson – Lie algebras, John Wiley and Sons, New York, London, Sydney.
3. V. S. Varadarajan – Lie groups, Lie algebras and their representations, Prentice Hall, New- Jersey.

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(4) ADVANCED FUNCTIONAL ANALYSIS

Bounded linear operators on Hilbert spaces, the adjoint of an operator, self adjoint operators, positive operators, properties of normal and unitary operators. One to one correspondence between projections on a Banach space and pairs of closed linear subspaces of the space, properties of orthogonal projections on Hilbert spaces. Spectral resolution of an operator on a finite dimensional Hilbert space H and the spectral theorem of a normal operator on H.

The structure of commutative Banach algebras - properties of the Gelfand mapping, the maximal ideal space, multiplicative functional and the maximal ideal. Applications of spectral radius formula. Involutions in Banach algebras, the Gelfand - Neumark theorem.

References:

1. G. F. Simmons – Introduction to Topology and Modern Analysis, Tata McGraw-Hill, New Delhi.
2. A. E. Taylor – Introduction to Functional Analysis, Wiley, New York, 1958.
3. A. Page and A. L. Brown – Elements of Functional Analysis.
4. George Bachman and Lawrence Narici – Functional Analysis, Dover Publications, Inc., Mineola, New York.
5. J. B. Conway – A course in Functional Analysis, GTM, Vol. 96., Springer, 1985.

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(5) ADVANCED GRAPH THEORY

Traversability - Eulerian graphs, Hamiltonian graphs.

Line Graphs - Some properties of line graphs, Characterization of line graphs, Special line graphs, Line graphs and traversability.

Factorization.

Planarity - Plane and planar graphs, Euler’s formula, Characterizations of planar graphs, Nonplanar graphs, Outerplanar graphs.

Colorability - the chromatic number, Five color theorem.

Matrices – The adjacency matrix, The incidence matrix, The cycle matrix.

Groups – The automorphism group of a graph, Operation on Permutation groups, The group of a composite graph, Graphs with a given group, Symmetric graphs, Highly symmetric graphs.
Domination Theory - Domination numbers, Some elementary properties.

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